Definitions

Make sure to run the code below -- but don't worry about the details, these are just definitions of the functions our demo will use

In[ ]:=

```
SetDirectory[NotebookDirectory[]];
Import["https://qtechtheory.org/QuESTlink.m"]
CreateDownloadedQuESTEnv[];
pswap[p1_, p2_] :=
  {{e^(-i p2), 0, 0, 0}, {0, 1/2 e^(-2 i p1+i p2) + 1/2 e^(2 i p1+i p2), 1/2 e^(-2 i p1+i p2) - 1/2 e^(2 i p1+i p2), 0},
   {0, 1/2 e^(-2 i p1+i p2) - 1/2 e^(2 i p1+i p2), 1/2 e^(-2 i p1+i p2) + 1/2 e^(2 i p1+i p2), 0}, {0, 0, 0, e^(-i p2)}};
ccphasemat[θ_] := {{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
   {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
   {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, e^(i θ/2)}};

gate[c_, n1_, n2_, typ_] := Switch[typ

  , "crx", {Ry_n1[π/2], Rz_n2[5 π/2], C_n1[Rx_n2[π]],
   Rz_n2[7 π/2], C_c[Rx_n2[5 π/2]], C_c[Rx_n1[π/2]], C_n1[Rx_n2[π]], Rz_n1[3 π/2],
   C_c[Rx_n1[3 π/2]], Ry_n1[7 π/2], Rz_n2[3 π/2], C_n2[Rx_n1[π]], Rz_c[7 π/4]}

  , "crxUE", {C_c[Rx_n1[3 π]], C_n1[Rx_n2[π]], Rz_n1[5 π/2],
   Rz_n2[13 π/4], C_n2[Rx_n1[π/2]], C_c[Rx_n2[3 π]], C_c[Rx_n1[π/2]], Rz_c[π/4]}
  , "crxOBSX", {C_n1[Rx_n2[π]], Rz_n1[5 π/2], Rz_n2[15 π/4],
   C_n2[Rx_n1[3 π/2]], C_c[Rx_n2[π]], C_c[Rx_n1[π/2]], Rz_c[-π/4]}

  , "xx", {Ry_c[7 π/2], Rz_n1[7 π/2], R[5 π/2, X_n1 X_n2], Rz_n1[7 π/4], Rz_n2[3 π/4], Ry_n1[π/2],
   R[7 π/2, X_c X_n2], Ry_n2[11 π/4], R[7 π/2, X_n1 X_n2], R[π/4, X_c X_n1], Rz_n2[π/4], R[5 π/2, X_c X_n2],
   Ry_c[5 π/2], Rz_n1[5 π/2], Ry_n2[7 π/4], R[7 π/2, X_n1 X_n2], Ry_n1[π/2], Rz_c[11 π/4]}

  , "xxUE", {Ry_c[7 π/2], Rz_n2[π/2], R[7 π/2, X_n1 X_n2], R[7 π/2, X_c X_n1], Ry_n1[5 π/4], Ry_n2[5 π/2],
   R[3 π/2, X_n1 X_n2], Rz_n1[3 π/4], R[11 π/4, X_c X_n2], R[π/2, X_c X_n1], Ry_c[π/2], Rz_c[5 π/4]}
  , "xxOBSX", {Rz_n2[-π/2], R[π/2, X_n1 X_n2], Rz_n1[3 π/4], Ry_n2[π/2], Ry_c[3 π/2],
```

$$R\left[\frac{7\pi}{2}, X_{n1} X_{n2}\right], R\left[\frac{5\pi}{4}, X_c X_{n2}\right], Ry_{n1}\left[\frac{3\pi}{4}\right], R\left[\frac{\pi}{2}, X_c X_{n1}\right], Ry_c\left[\frac{\pi}{2}\right], Rz_c\left[\frac{5\pi}{4}\right]\}$$

, "pswap",
$$\{Rx_{n1}\left[\frac{\pi}{2}\right], Rx_{n2}\left[\frac{5\pi}{2}\right], pSWAP_{n1,n2}\left[\frac{31\pi}{8}, \frac{3\pi}{4}\right], pSWAP_{n2,c}\left[\frac{7\pi}{2}, \frac{7\pi}{4}\right], Rz_{n1}\left[\frac{5\pi}{2}\right], Ry_{n1}\left[\frac{7\pi}{2}\right],$$
$$Ry_{n2}\left[\frac{\pi}{2}\right], pSWAP_{n1,n2}\left[\frac{\pi}{8}, \frac{5\pi}{2}\right], pSWAP_{n1,c}\left[\frac{\pi}{4}, \frac{5\pi}{2}\right], Rz_{n2}\left[\frac{7\pi}{2}\right], Ry_{n2}\left[\frac{5\pi}{2}\right], Ry_c\left[\frac{\pi}{2}\right],$$
$$pSWAP_{n2,c}\left[\frac{19\pi}{8}, \frac{\pi}{2}\right], pSWAP_{n1,c}\left[\frac{\pi}{4}, \pi\right], Rz_{n1}\left[\frac{\pi}{2}\right], Ry_{n1}\left[\frac{\pi}{2}\right], Ry_{n2}\left[\frac{7\pi}{2}\right], Rz_c\left[\frac{13\pi}{4}\right]\}$$

, "pswapUE", $\{Rz_{n2}\left[\frac{3\pi}{2}\right], pSWAP_{n1,n2}\left[\frac{19\pi}{8}, \frac{159\pi}{200}\right],$
$$pSWAP_{n2,c}\left[n1, \frac{13\pi}{8}\right], pSWAP_{n1,c}\left[n1, \frac{23\pi}{8}\right], Ry_{n2}\left[\frac{7\pi}{2}\right], pSWAP_{n1,n2}\left[\frac{15\pi}{4}, \frac{7\pi}{2}\right],$$
$$Rz_{n1}\left[\frac{7\pi}{2}\right], Ry_{n1}\left[\frac{\pi}{2}\right], pSWAP_{n1,c}\left[\frac{7\pi}{2}, \frac{31\pi}{8}\right], Rz_c\left[\frac{7\pi}{4}\right]\}$$
, "pswapOBSX", $\{pSWAP_{0,1}\left[\frac{5\pi}{8}, \frac{5\pi}{2}\right], pSWAP_{1,2}\left[\frac{\pi}{2}, \frac{13\pi}{4}\right], Rz_{n1}\left[\frac{\pi}{2}\right],$
$$Ry_{n1}\left[\frac{3\pi}{2}\right], Ry_{n2}\left[\frac{5\pi}{2}\right], pSWAP_{0,1}\left[\frac{25\pi}{8}, 0\right], pSWAP_{0,2}\left[\pi, \frac{15\pi}{4}\right], Rz_c\left[\frac{13\pi}{4}\right]\}$$

, "cz", $\{Rx_{n2}\left[\frac{7\pi}{2}\right], C_{n1}[Rz_{n2}[3\pi]], Ry_{n2}\left[\frac{3\pi}{4}\right], Rz_{n2}[3\pi],$
$$C_c[Rz_{n2}[\pi]], Ry_{n1}\left[\frac{5\pi}{2}\right], Ry_{n2}\left[\frac{7\pi}{4}\right], C_{n2}[Rz_{n1}[\pi]], C_c\left[Rz_{n1}\left[\frac{5\pi}{2}\right]\right],$$
$$Rx_{n1}\left[\frac{3\pi}{2}\right], Rx_{n2}\left[\frac{3\pi}{4}\right], Rz_{n1}\left[\frac{5\pi}{2}\right], C_c[Rz_{n2}[\pi]], Rx_{n1}[\pi], Rx_{n2}\left[\frac{\pi}{4}\right],$$
$$C_{n2}[Rz_{n1}[\pi]], Ry_{n2}\left[\frac{7\pi}{2}\right], Rx_{n1}\left[\frac{\pi}{2}\right], Rx_c[n1], Rz_{n1}\left[\frac{7\pi}{2}\right], Rz_{n2}\left[\frac{\pi}{2}\right], Rz_c\left[\frac{3\pi}{4}\right]\}$$

, "czUE", $\{Ry_{n1}\left[\frac{\pi}{2}\right], C_{n2}[Rz_{n1}[-\pi]], C_c[Rz_{n1}[\pi]], Rx_{n1}\left[\frac{3\pi}{4}\right], Rx_{n2}\left[\frac{5\pi}{2}\right],$
$$C_{n2}[Rz_{n1}[-\pi]], Rx_{n1}\left[\frac{11\pi}{4}\right], C_c\left[Rz_{n2}\left[\frac{7\pi}{2}\right]\right], C_c[Rz_{n1}[-\pi]], Rz_c\left[\frac{9\pi}{4}\right]\}$$
, "czOBSX", $\{Ry_{n1}\left[\frac{7\pi}{2}\right], C_{n1}[Rz_{n2}[\pi]], Ry_{n2}\left[\frac{7\pi}{2}\right], Rx_{n1}\left[\frac{3\pi}{2}\right],$
$$C_{n2}\left[Rz_{n1}\left[\frac{3\pi}{2}\right]\right], C_c\left[Rz_{n2}\left[\frac{5\pi}{2}\right]\right], Rx_{n1}\left[\frac{\pi}{2}\right], C_c[Rz_{n1}[\pi]], Rz_c\left[\frac{15\pi}{4}\right]\}$$

, "xxx", $\{Ry_c\left[\frac{\pi}{2}\right], R\left[\frac{11\pi}{4}, X_{n1} X_{n2}\right], R\left[\frac{\pi}{4}, X_c X_{n1} X_{n2}\right], Ry_{n1}\left[\frac{3\pi}{2}\right], Ry_{n2}\left[\frac{5\pi}{2}\right],$
$$R\left[\frac{\pi}{4}, X_{n1} X_{n2}\right], R\left[\frac{3\pi}{4}, X_c X_{n1} X_{n2}\right], Ry_{n1}\left[\frac{\pi}{2}\right], Ry_{n2}\left[\frac{3\pi}{2}\right], Rz_{n1}\left[\frac{5\pi}{2}\right], Rz_{n2}\left[\frac{3\pi}{2}\right],$$
$$R\left[\frac{5\pi}{4}, X_{n1} X_{n2}\right], R\left[\frac{15\pi}{4}, X_c X_{n1} X_{n2}\right], Ry_c\left[\frac{3\pi}{2}\right], Rz_{n1}\left[\frac{7\pi}{2}\right], Rz_{n2}\left[\frac{\pi}{2}\right], Rz_c\left[\frac{11\pi}{4}\right]\}$$

, "xxxUE", $\{Ry_c\left[\frac{\pi}{2}\right], R\left[\frac{3\pi}{4}, X_c X_{n1} X_{n2}\right], Ry_{n1}\left[\frac{\pi}{2}\right], Ry_{n2}\left[\frac{7\pi}{2}\right], R\left[\frac{13\pi}{4}, X_c X_{n1} X_{n2}\right],$
$$Rz_{n1}\left[\frac{5\pi}{2}\right], Rz_{n2}\left[\frac{\pi}{2}\right], R\left[\frac{11\pi}{4}, X_c X_{n1} X_{n2}\right], Ry_c\left[\frac{3\pi}{2}\right], Rz_c\left[\frac{13\pi}{4}\right]\}$$

```
  , "xxxOBSX", {Ry_c[π/2], R[5π/4, X_c X_n1 X_n2], Ry_n2[π/2], Rz_n1[3π/2], R[π/2, X_n1 X_n2],
   Ry_n1[15π/4], Rz_n2[15π/4], R[π/2, X_c X_n1 X_n2], Ry_c[3π/2], Rz_c[9π/4]}

  , "ccphase", {Rx_n2[5π/2], C_n2[Rz_n1[π]], Ry_n2[π/2], Rx_n1[π/2],
   CCPhase_c,n1,n2[2π], Ry_n2[7π/2], Rx_n1[7π/2], C_n2[Rz_n1[3π]], Rx_n2[3π/2]}
  , "ccphaseUE", {Ry_n1[7π/2], C_n2[Rz_n1[π]], Ry_n1[5π/2], Rx_n2[7π/2], CCPhase_c,n1,n2[2π]}
  , "ccphaseOBSX", {Ry_n2[5π/2], C_n2[Rz_n1[π]],
   C_c[Rz_n2[-π]], Ry_n1[π/2], Rx_n2[5π/2], CCPhase_c,n1,n2[2π], Rz_c[π/2]}
]
```

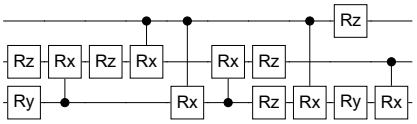
1) Type A and Type B recompilations

Type A recompilation: fully equivalent recompilation of the elementary controlled-SWAP gate
Type B recompilation: the elementary controlled-SWAP gate was recompiled up to an SU(4) freedom on the two swapped qubits

In[•]:=
```
ρ = CreateDensityQureg[3];
gateCanDo = {"crx", "crxUE", "xx", "xxUE", "pswap",
    "pswapUE", "cz", "czUE", "xxx", "xxxUE", "ccphase", "ccphaseUE"};
dat = {};
Do[
 gateTyp = gateCanDo[[k]];
 cSWAPCirc = gate[2, 0, 1, gateTyp];
 circ = Flatten@{H2, cSWAPCirc, H2} /. {pSWAPn1_,n2_[p1_, p2_] :> Un1,n2[pswap[p1, p2]],
    CCPhasec_,n1_,n2_[p1_] :> Uc,n1,n2[ccphasemat[p1]]};
 twoQubitGateCount = Count[cSWAPCirc, R[__] | U__[__] |
    C__[__] | CCPhase__[__] | pSWAP__[__]];
 rand = With[{r = RandomComplex[{-1 - i, 1 + i}, {2, 2}]},
      r . r† / Tr[r . r†]];
 SetQuregMatrix[ρ, KroneckerProduct[{{1, 0}, {0, 0}}, KroneckerProduct[rand, rand]]];
 ApplyCircuit[circ, ρ];
 circPic =
  DrawCircuit[cSWAPCirc /. {CCPhasec_,n1_,n2_[__] :> Cc,n1[Zn2], pSWAPn1_,n2_[__] :> pSWn1,n2},
   ImageSize → {Automatic, 60}, FontFamily → "Times New Roman"];
 AppendTo[dat, {circPic, twoQubitGateCount, Length@cSWAPCirc - twoQubitGateCount}];
 Print["Gate Type: ", gateTyp, "\n#2-qubit gates:  " , twoQubitGateCount,
  "\n#1-qubit gates:  ", Length@cSWAPCirc - twoQubitGateCount,
  "(-1) by removing the Z gate at the end\n",
  circPic
  , "\n", gate[c, n1, n2, gateTyp],
   "\nis the circuit correct:   "
  , Chop[2 CalcProbOfOutcome[ρ, 2, 0] - 1 - Tr[rand.rand]] == 0, "\n\n"
 ];
 , {k, 1, Length@gateCanDo}]

DestroyAllQuregs[];
```

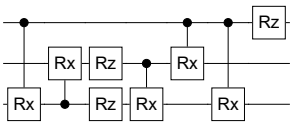
```
Gate Type: crx
#2-qubit gates:  6
#1-qubit gates:  7(-1) by removing the Z gate at the end
```



$$\left\{\mathrm{Ry}_{n1}\left[\frac{\pi}{2}\right],\ \mathrm{Rz}_{n2}\left[\frac{5\pi}{2}\right],\ \mathrm{C}_{n1}[\mathrm{Rx}_{n2}[\pi]],\ \mathrm{Rz}_{n2}\left[\frac{7\pi}{2}\right],\ \mathrm{C}_{c}\left[\mathrm{Rx}_{n2}\left[\frac{5\pi}{2}\right]\right],\ \mathrm{C}_{c}\left[\mathrm{Rx}_{n1}\left[\frac{\pi}{2}\right]\right],\right.$$
$$\left.\mathrm{C}_{n1}[\mathrm{Rx}_{n2}[\pi]],\ \mathrm{Rz}_{n1}\left[\frac{3\pi}{2}\right],\ \mathrm{C}_{c}\left[\mathrm{Rx}_{n1}\left[\frac{3\pi}{2}\right]\right],\ \mathrm{Ry}_{n1}\left[\frac{7\pi}{2}\right],\ \mathrm{Rz}_{n2}\left[\frac{3\pi}{2}\right],\ \mathrm{C}_{n2}[\mathrm{Rx}_{n1}[\pi]],\ \mathrm{Rz}_{c}\left[\frac{7\pi}{4}\right]\right\}$$

```
is the circuit correct:   True
```

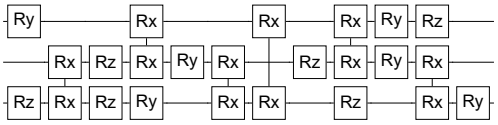
Gate Type: crxUE
#2-qubit gates: 5
#1-qubit gates: 3(-1) by removing the Z gate at the end



$$\left\{C_c[Rx_{n1}[3\pi]],\ C_{n1}[Rx_{n2}[\pi]],\ Rz_{n1}\left[\frac{5\pi}{2}\right],\right.$$
$$\left.Rz_{n2}\left[\frac{13\pi}{4}\right],\ C_{n2}\left[Rx_{n1}\left[\frac{\pi}{2}\right]\right],\ C_c[Rx_{n2}[3\pi]],\ C_c\left[Rx_{n1}\left[\frac{\pi}{2}\right]\right],\ Rz_c\left[\frac{\pi}{4}\right]\right\}$$

is the circuit correct: True

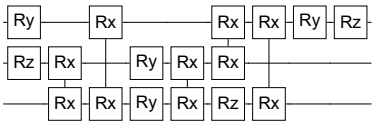
Gate Type: xx
#2-qubit gates: 6
#1-qubit gates: 12(-1) by removing the Z gate at the end



$$\left\{Ry_c\left[\frac{7\pi}{2}\right],\ Rz_{n1}\left[\frac{7\pi}{2}\right],\ R\left[\frac{5\pi}{2},\ X_{n1}X_{n2}\right],\ Rz_{n1}\left[\frac{7\pi}{4}\right],\ Rz_{n2}\left[\frac{3\pi}{4}\right],\ Ry_{n1}\left[\frac{\pi}{2}\right],\right.$$
$$R\left[\frac{7\pi}{2},\ X_c X_{n2}\right],\ Ry_{n2}\left[\frac{11\pi}{4}\right],\ R\left[\frac{7\pi}{2},\ X_{n1}X_{n2}\right],\ R\left[\frac{\pi}{4},\ X_c X_{n1}\right],\ Rz_{n2}\left[\frac{\pi}{4}\right],\ R\left[\frac{5\pi}{2},\ X_c X_{n2}\right],$$
$$\left.Ry_c\left[\frac{5\pi}{2}\right],\ Rz_{n1}\left[\frac{5\pi}{2}\right],\ Ry_{n2}\left[\frac{7\pi}{4}\right],\ R\left[\frac{7\pi}{2},\ X_{n1}X_{n2}\right],\ Ry_{n1}\left[\frac{\pi}{2}\right],\ Rz_c\left[\frac{11\pi}{4}\right]\right\}$$

is the circuit correct: True

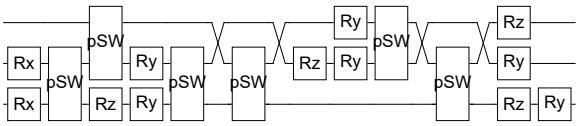
Gate Type: xxUE
#2-qubit gates: 5
#1-qubit gates: 7(-1) by removing the Z gate at the end



$$\left\{Ry_c\left[\frac{7\pi}{2}\right],\ Rz_{n2}\left[\frac{\pi}{2}\right],\ R\left[\frac{7\pi}{2},\ X_{n1}X_{n2}\right],\ R\left[\frac{7\pi}{2},\ X_c X_{n1}\right],\ Ry_{n1}\left[\frac{5\pi}{4}\right],\ Ry_{n2}\left[\frac{5\pi}{2}\right],\right.$$
$$\left.R\left[\frac{3\pi}{2},\ X_{n1}X_{n2}\right],\ Rz_{n1}\left[\frac{3\pi}{4}\right],\ R\left[\frac{11\pi}{4},\ X_c X_{n2}\right],\ R\left[\frac{\pi}{2},\ X_c X_{n1}\right],\ Ry_c\left[\frac{\pi}{2}\right],\ Rz_c\left[\frac{5\pi}{4}\right]\right\}$$

is the circuit correct: True

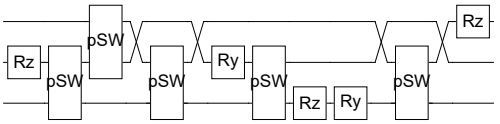
Gate Type: pswap
#2-qubit gates: 6
#1-qubit gates: 12(-1) by removing the Z gate at the end



$$\left\{\mathrm{Rx}_{n1}\left[\frac{\pi}{2}\right],\ \mathrm{Rx}_{n2}\left[\frac{5\pi}{2}\right],\ \mathrm{pSWAP}_{n1,n2}\left[\frac{31\pi}{8},\ \frac{3\pi}{4}\right],\ \mathrm{pSWAP}_{n2,c}\left[\frac{7\pi}{2},\ \frac{7\pi}{4}\right],\ \mathrm{Rz}_{n1}\left[\frac{5\pi}{2}\right],\ \mathrm{Ry}_{n1}\left[\frac{7\pi}{2}\right],\right.$$
$$\mathrm{Ry}_{n2}\left[\frac{\pi}{2}\right],\ \mathrm{pSWAP}_{n1,n2}\left[\frac{\pi}{8},\ \frac{5\pi}{2}\right],\ \mathrm{pSWAP}_{n1,c}\left[\frac{\pi}{4},\ \frac{5\pi}{2}\right],\ \mathrm{Rz}_{n2}\left[\frac{7\pi}{2}\right],\ \mathrm{Ry}_{n2}\left[\frac{5\pi}{2}\right],\ \mathrm{Ry}_{c}\left[\frac{\pi}{2}\right],$$
$$\left.\mathrm{pSWAP}_{n2,c}\left[\frac{19\pi}{8},\ \frac{\pi}{2}\right],\ \mathrm{pSWAP}_{n1,c}\left[\frac{\pi}{4},\ \pi\right],\ \mathrm{Rz}_{n1}\left[\frac{\pi}{2}\right],\ \mathrm{Ry}_{n1}\left[\frac{\pi}{2}\right],\ \mathrm{Ry}_{n2}\left[\frac{7\pi}{2}\right],\ \mathrm{Rz}_{c}\left[\frac{13\pi}{4}\right]\right\}$$

is the circuit correct: True

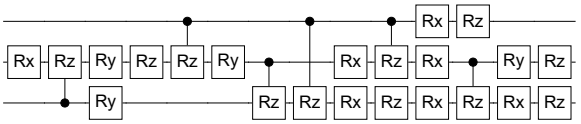
Gate Type: pswapUE
#2-qubit gates: 5
#1-qubit gates: 5(-1) by removing the Z gate at the end



$$\left\{\mathrm{Rz}_{n2}\left[\frac{3\pi}{2}\right],\ \mathrm{pSWAP}_{n1,n2}\left[\frac{19\pi}{8},\ \frac{159\pi}{200}\right],\ \mathrm{pSWAP}_{n2,c}\left[\mathrm{n1},\ \frac{13\pi}{8}\right],\ \mathrm{pSWAP}_{n1,c}\left[\mathrm{n1},\ \frac{23\pi}{8}\right],\right.$$
$$\left.\mathrm{Ry}_{n2}\left[\frac{7\pi}{2}\right],\ \mathrm{pSWAP}_{n1,n2}\left[\frac{15\pi}{4},\ \frac{7\pi}{2}\right],\ \mathrm{Rz}_{n1}\left[\frac{7\pi}{2}\right],\ \mathrm{Ry}_{n1}\left[\frac{\pi}{2}\right],\ \mathrm{pSWAP}_{n1,c}\left[\frac{7\pi}{2},\ \frac{31\pi}{8}\right],\ \mathrm{Rz}_{c}\left[\frac{7\pi}{4}\right]\right\}$$

is the circuit correct: True

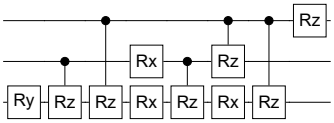
Gate Type: cz
#2-qubit gates: 6
#1-qubit gates: 16(-1) by removing the Z gate at the end



$$\left\{\mathrm{Rx}_{n2}\left[\frac{7\pi}{2}\right],\ \mathrm{C}_{n1}\left[\mathrm{Rz}_{n2}\left[3\pi\right]\right],\ \mathrm{Ry}_{n2}\left[\frac{3\pi}{4}\right],\ \mathrm{Rz}_{n2}\left[3\pi\right],\ \mathrm{C}_{c}\left[\mathrm{Rz}_{n2}\left[\pi\right]\right],\ \mathrm{Ry}_{n1}\left[\frac{5\pi}{2}\right],\ \mathrm{Ry}_{n2}\left[\frac{7\pi}{4}\right],\right.$$
$$\mathrm{C}_{n2}\left[\mathrm{Rz}_{n1}\left[\pi\right]\right],\ \mathrm{C}_{c}\left[\mathrm{Rz}_{n1}\left[\frac{5\pi}{2}\right]\right],\ \mathrm{Rx}_{n1}\left[\frac{3\pi}{2}\right],\ \mathrm{Rx}_{n2}\left[\frac{3\pi}{4}\right],\ \mathrm{Rz}_{n1}\left[\frac{5\pi}{2}\right],\ \mathrm{C}_{c}\left[\mathrm{Rz}_{n2}\left[\pi\right]\right],\ \mathrm{Rx}_{n1}\left[\pi\right],$$
$$\left.\mathrm{Rx}_{n2}\left[\frac{\pi}{4}\right],\ \mathrm{C}_{n2}\left[\mathrm{Rz}_{n1}\left[\pi\right]\right],\ \mathrm{Ry}_{n2}\left[\frac{7\pi}{2}\right],\ \mathrm{Rx}_{n1}\left[\frac{\pi}{2}\right],\ \mathrm{Rx}_{c}\left[\mathrm{n1}\right],\ \mathrm{Rz}_{n1}\left[\frac{7\pi}{2}\right],\ \mathrm{Rz}_{n2}\left[\frac{\pi}{2}\right],\ \mathrm{Rz}_{c}\left[\frac{3\pi}{4}\right]\right\}$$

is the circuit correct: True

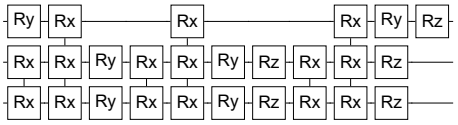
Gate Type: czUE
#2-qubit gates: 5
#1-qubit gates: 5(-1) by removing the Z gate at the end



$$\left\{\text{Ry}_{n1}\left[\frac{\pi}{2}\right],\ \text{C}_{n2}[\text{Rz}_{n1}[-\pi]],\ \text{C}_{c}[\text{Rz}_{n1}[\pi]],\ \text{Rx}_{n1}\left[\frac{3\pi}{4}\right],\ \text{Rx}_{n2}\left[\frac{5\pi}{2}\right],\right.$$
$$\left.\text{C}_{n2}[\text{Rz}_{n1}[-\pi]],\ \text{Rx}_{n1}\left[\frac{11\pi}{4}\right],\ \text{C}_{c}\left[\text{Rz}_{n2}\left[\frac{7\pi}{2}\right]\right],\ \text{C}_{c}[\text{Rz}_{n1}[-\pi]],\ \text{Rz}_{c}\left[\frac{9\pi}{4}\right]\right\}$$

is the circuit correct: True

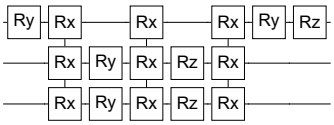
Gate Type: xxx
#2-qubit gates: 6
#1-qubit gates: 11(-1) by removing the Z gate at the end



$$\left\{\text{Ry}_{c}\left[\frac{\pi}{2}\right],\ \text{R}\left[\frac{11\pi}{4},\ \text{X}_{n1}\,\text{X}_{n2}\right],\ \text{R}\left[\frac{\pi}{4},\ \text{X}_{c}\,\text{X}_{n1}\,\text{X}_{n2}\right],\ \text{Ry}_{n1}\left[\frac{3\pi}{2}\right],\ \text{Ry}_{n2}\left[\frac{5\pi}{2}\right],\right.$$
$$\text{R}\left[\frac{\pi}{4},\ \text{X}_{n1}\,\text{X}_{n2}\right],\ \text{R}\left[\frac{3\pi}{4},\ \text{X}_{c}\,\text{X}_{n1}\,\text{X}_{n2}\right],\ \text{Ry}_{n1}\left[\frac{\pi}{2}\right],\ \text{Ry}_{n2}\left[\frac{3\pi}{2}\right],\ \text{Rz}_{n1}\left[\frac{5\pi}{2}\right],\ \text{Rz}_{n2}\left[\frac{3\pi}{2}\right],$$
$$\left.\text{R}\left[\frac{5\pi}{4},\ \text{X}_{n1}\,\text{X}_{n2}\right],\ \text{R}\left[\frac{15\pi}{4},\ \text{X}_{c}\,\text{X}_{n1}\,\text{X}_{n2}\right],\ \text{Ry}_{c}\left[\frac{3\pi}{2}\right],\ \text{Rz}_{n1}\left[\frac{7\pi}{2}\right],\ \text{Rz}_{n2}\left[\frac{\pi}{2}\right],\ \text{Rz}_{c}\left[\frac{11\pi}{4}\right]\right\}$$

is the circuit correct: True

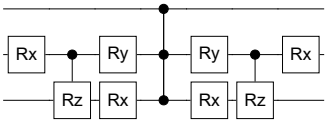
Gate Type: xxxUE
#2-qubit gates: 3
#1-qubit gates: 7(-1) by removing the Z gate at the end



$$\left\{\text{Ry}_{c}\left[\frac{\pi}{2}\right],\ \text{R}\left[\frac{3\pi}{4},\ \text{X}_{c}\,\text{X}_{n1}\,\text{X}_{n2}\right],\ \text{Ry}_{n1}\left[\frac{\pi}{2}\right],\ \text{Ry}_{n2}\left[\frac{7\pi}{2}\right],\right.$$
$$\left.\text{R}\left[\frac{13\pi}{4},\ \text{X}_{c}\,\text{X}_{n1}\,\text{X}_{n2}\right],\ \text{Rz}_{n1}\left[\frac{5\pi}{2}\right],\ \text{Rz}_{n2}\left[\frac{\pi}{2}\right],\ \text{R}\left[\frac{11\pi}{4},\ \text{X}_{c}\,\text{X}_{n1}\,\text{X}_{n2}\right],\ \text{Ry}_{c}\left[\frac{3\pi}{2}\right],\ \text{Rz}_{c}\left[\frac{13\pi}{4}\right]\right\}$$

is the circuit correct: True

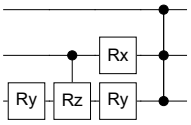
```
Gate Type: ccphase
#2-qubit gates:  3
#1-qubit gates:  6(-1) by removing the Z gate at the end
```



$$\left\{Rx_{n2}\left[\frac{5\pi}{2}\right], C_{n2}[Rz_{n1}[\pi]], Ry_{n2}\left[\frac{\pi}{2}\right], Rx_{n1}\left[\frac{\pi}{2}\right],\right.$$
$$\left. CCPhase_{c,n1,n2}[2\pi], Ry_{n2}\left[\frac{7\pi}{2}\right], Rx_{n1}\left[\frac{7\pi}{2}\right], C_{n2}[Rz_{n1}[3\pi]], Rx_{n2}\left[\frac{3\pi}{2}\right]\right\}$$

```
is the circuit correct:   True
```

```
Gate Type: ccphaseUE
#2-qubit gates:  2
#1-qubit gates:  3(-1) by removing the Z gate at the end
```



$$\left\{Ry_{n1}\left[\frac{7\pi}{2}\right], C_{n2}[Rz_{n1}[\pi]], Ry_{n1}\left[\frac{5\pi}{2}\right], Rx_{n2}\left[\frac{7\pi}{2}\right], CCPhase_{c,n1,n2}[2\pi]\right\}$$

```
is the circuit correct:   True
```

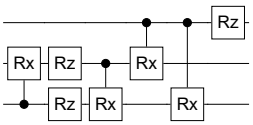
1) Type C recompilations

Type C recompilation: the product of the elementary controlled-SWAP gate and the controlled X observable was recompiled up to an SU(4) freedom on the two swapped qubits

```
In[•]:= ρ = CreateDensityQureg[3];
gateCanDo = {"crxOBSX", "xxOBSX", "czOBSX", "ccphaseOBSX", "pswapOBSX", "xxxOBSX"};
dat = {};
Do[
 gateTyp = gateCanDo[[k]];
 cSWAPCirc = gate[2, 0, 1, gateTyp];
 circ = Flatten@{H2, cSWAPCirc, H2} /. {pSWAPn1_,n2_[p1_, p2_] :↦ Un1,n2[pswap[p1, p2]],
    CCPhasec_,n1_,n2_[p1_] :↦ Uc,n1,n2[ccphasemat[p1]]};
 twoQubitGateCount = Count[cSWAPCirc, R[__] | U__[__] |
    C__[__] | CCPhase__[__] | pSWAP__[__]];
 rand = With[{r = RandomComplex[{-1 - ⅈ, 1 + ⅈ}, {2, 2}]},
       r . r† / Tr[r . r†]];
 SetQuregMatrix[ρ, KroneckerProduct[{{1, 0}, {0, 0}}, KroneckerProduct[rand, rand]]];
 ApplyCircuit[circ, ρ];
 circPic =
  DrawCircuit[cSWAPCirc /. {CCPhasec_,n1_,n2_[__] :↦ Cc,n1[Zn2], pSWAPn1_,n2_[__] :↦ pSWn1,n2},
   ImageSize → {Automatic, 60}, FontFamily → "Times New Roman"];
 AppendTo[dat, {circPic, twoQubitGateCount, Length@cSWAPCirc - twoQubitGateCount}];
 Print["Gate Type: ", gateTyp, "\n#2-qubit gates:  " , twoQubitGateCount,
  "\n#1-qubit gates:  ", Length@cSWAPCirc - twoQubitGateCount,
  "(-1) by removing the Z gate at the end\n",
  circPic
  , "\n", gate[c, n1, n2, gateTyp],
   "\nis the circuit correct:   "
  , Chop[2 CalcProbOfOutcome[ρ, 2, 0] - 1 - Tr[rand.rand.PauliMatrix[1]]] == 0, "\n\n"
 ];
 , {k, 1, Length@gateCanDo}]

DestroyAllQuregs[];
```

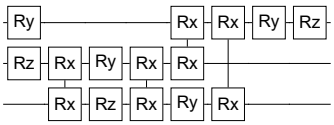
```
Gate Type: crxOBSX
#2-qubit gates:  4
#1-qubit gates:  3(-1) by removing the Z gate at the end
```



$$\left\{C_{n1}[Rx_{n2}[\pi]],\ Rz_{n1}\left[\frac{5\pi}{2}\right],\ Rz_{n2}\left[\frac{15\pi}{4}\right],\ C_{n2}\left[Rx_{n1}\left[\frac{3\pi}{2}\right]\right],\ C_c[Rx_{n2}[\pi]],\ C_c\left[Rx_{n1}\left[\frac{\pi}{2}\right]\right],\ Rz_c\left[-\frac{\pi}{4}\right]\right\}$$

```
is the circuit correct:   True
```

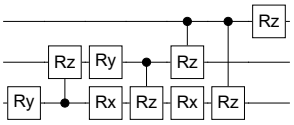
Gate Type: xxOBSX
#2-qubit gates: 4
#1-qubit gates: 7(-1) by removing the Z gate at the end



$$\left\{\text{Rz}_{n2}\left[-\frac{\pi}{2}\right],\ \text{R}\left[\frac{\pi}{2},\ \text{X}_{n1}\,\text{X}_{n2}\right],\ \text{Rz}_{n1}\left[\frac{3\,\pi}{4}\right],\ \text{Ry}_{n2}\left[\frac{\pi}{2}\right],\ \text{Ry}_{c}\left[\frac{3\,\pi}{2}\right],\right.$$
$$\left.\text{R}\left[\frac{7\,\pi}{2},\ \text{X}_{n1}\,\text{X}_{n2}\right],\ \text{R}\left[\frac{5\,\pi}{4},\ \text{X}_{c}\,\text{X}_{n2}\right],\ \text{Ry}_{n1}\left[\frac{3\,\pi}{4}\right],\ \text{R}\left[\frac{\pi}{2},\ \text{X}_{c}\,\text{X}_{n1}\right],\ \text{Ry}_{c}\left[\frac{\pi}{2}\right],\ \text{Rz}_{c}\left[\frac{5\,\pi}{4}\right]\right\}$$

is the circuit correct: True

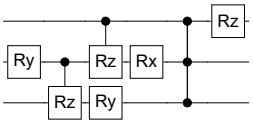
Gate Type: czOBSX
#2-qubit gates: 4
#1-qubit gates: 5(-1) by removing the Z gate at the end



$$\left\{\text{Ry}_{n1}\left[\frac{7\,\pi}{2}\right],\ \text{C}_{n1}[\text{Rz}_{n2}[\pi]],\ \text{Ry}_{n2}\left[\frac{7\,\pi}{2}\right],\ \text{Rx}_{n1}\left[\frac{3\,\pi}{2}\right],\right.$$
$$\left.\text{C}_{n2}\left[\text{Rz}_{n1}\left[\frac{3\,\pi}{2}\right]\right],\ \text{C}_{c}\left[\text{Rz}_{n2}\left[\frac{5\,\pi}{2}\right]\right],\ \text{Rx}_{n1}\left[\frac{\pi}{2}\right],\ \text{C}_{c}[\text{Rz}_{n1}[\pi]],\ \text{Rz}_{c}\left[\frac{15\,\pi}{4}\right]\right\}$$

is the circuit correct: True

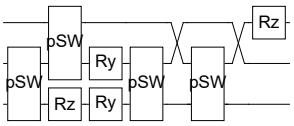
Gate Type: ccphaseOBSX
#2-qubit gates: 3
#1-qubit gates: 4(-1) by removing the Z gate at the end



$$\left\{\text{Ry}_{n2}\left[\frac{5\,\pi}{2}\right],\ \text{C}_{n2}[\text{Rz}_{n1}[\pi]],\ \text{C}_{c}[\text{Rz}_{n2}[-\pi]],\ \text{Ry}_{n1}\left[\frac{\pi}{2}\right],\ \text{Rx}_{n2}\left[\frac{5\,\pi}{2}\right],\ \text{CCPhase}_{c,n1,n2}[2\,\pi],\ \text{Rz}_{c}\left[\frac{\pi}{2}\right]\right\}$$

is the circuit correct: True

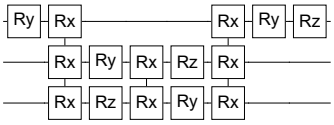
Gate Type: pswapOBSX
#2-qubit gates: 4
#1-qubit gates: 4(-1) by removing the Z gate at the end



$$\left\{\text{pSWAP}_{0,1}\left[\frac{5\pi}{8},\ \frac{5\pi}{2}\right],\ \text{pSWAP}_{1,2}\left[\frac{\pi}{2},\ \frac{13\pi}{4}\right],\ \text{Rz}_{n1}\left[\frac{\pi}{2}\right],\right.$$
$$\left.\text{Ry}_{n1}\left[\frac{3\pi}{2}\right],\ \text{Ry}_{n2}\left[\frac{5\pi}{2}\right],\ \text{pSWAP}_{0,1}\left[\frac{25\pi}{8},\ 0\right],\ \text{pSWAP}_{0,2}\left[\pi,\ \frac{15\pi}{4}\right],\ \text{Rz}_{c}\left[\frac{13\pi}{4}\right]\right\}$$

is the circuit correct: True

Gate Type: xxxOBSX
#2-qubit gates: 3
#1-qubit gates: 7(-1) by removing the Z gate at the end



$$\left\{\text{Ry}_{c}\left[\frac{\pi}{2}\right],\ \text{R}\left[\frac{5\pi}{4},\ \text{X}_{c}\,\text{X}_{n1}\,\text{X}_{n2}\right],\ \text{Ry}_{n2}\left[\frac{\pi}{2}\right],\ \text{Rz}_{n1}\left[\frac{3\pi}{2}\right],\right.$$
$$\left.\text{R}\left[\frac{\pi}{2},\ \text{X}_{n1}\,\text{X}_{n2}\right],\ \text{Ry}_{n1}\left[\frac{15\pi}{4}\right],\ \text{Rz}_{n2}\left[\frac{15\pi}{4}\right],\ \text{R}\left[\frac{\pi}{2},\ \text{X}_{c}\,\text{X}_{n1}\,\text{X}_{n2}\right],\ \text{Ry}_{c}\left[\frac{3\pi}{2}\right],\ \text{Rz}_{c}\left[\frac{9\pi}{4}\right]\right\}$$

is the circuit correct: True